**MODERN COLLEGE OF ARTS,SCI. & COMM. PUNE-05.**

**DEPARTMENT OF STATISTICS. (Autonomous)**

**M.Sc.( I )- ST-15**

**EXPT.NO. (9A) and (9B)**

**TITLE : Model sampling from bivariate probability distribution and computation of probabilities.**

1. Draw a random sample of size 18 from the bivariate normal distribution

( 2.8, 11.0, 0.16, 100,0.6).

2. Generate a random sample of size 20 from the Bivariate Poission distribution with

Parameter θ1=10, θ2=5, θ12=6 . Compute mean and variance for given sample.

3. Generate a random sample of size 20 from the bivariate exponential distribution with failure rates λ1=3, λ2=12, λ3=10 . Compute mean and variance for given sample.

4. Let (X , Y) has bivariate normal distribution with parameters μ1 = 3, μ2 = 1, σ12 = 16,

σ22 = 25 and ρ = 3/5. Determine the following probability.

(a) P(3<Y<8) (b) P(3<Y<8/ X= 8) (c) P ( -3<X<3) (d) P ( -3<X<3 / Y= -4)

5. .Let ( X, Y) have a bivariate normal distribution with parameter μ1 = 5, μ2 = 10,

σ12 = 1, σ22 = 25 and ρ >0. If P (4<Y<16 / X=5) = 0.954 determine ρ .

6. The life of a tube (X1) and the filament diameter (X2) are distributed as a bivariate normal random variable with parameter μ1= 2000 hrs, μ2 = 0.10 inch , = 2500 h2 and =0.01inch2 and =0.87. The quality control manager wishes to determine the life of each tube by measuring the filament diameter. If a filament diameter is 0.098, what is the probability that the tube will last for 1950 hours? 2122

7. If (X1, X2) ~ Bivariate exponential distribution with failure rates λ1=3, λ2=9, λ3=10 then compute marginal distribution of X1 and X2, hence find

i) E(X1) ii) E(X2) iii) V(X1) iv) V(X2) v) Correlation(X1, X2)

8. Suppose that an instrument has two components A and B with life time ( in minutes) X1 and X2. (X1,X2) ~ Bivariate exponential distribution (λ1=8, λ2=10, λ3=14). Calculate

i) P [first component fail before 10 minutes]

ii) P [Second component survive at least for 20 minutes]

9. A system consist of 10 component, the life times of ith component is Yi are i.i.d exponentially distributed with failure rate λi =20 hours i=1,2,…….10 per hours then calculate P[Y1≤ min(Y2,……..,Y10)]

10. If (X1, X2) ~ Bivariate Poisson distribution with parameter θ1=8, θ2=6, θ12=4 then compute marginal distribution of X1 and X2, hence find

i) E(X1) and E(X2) ii) V(X1) and V(X2) iii) Correlation(X1, X2)

iv) P(X1=5/X2=6)

11. If (X1, X2) ~ Bivariate Poisson distribution with parameter θ1=3, θ2=2, θ12=8 then

Compute (i) P[X1>4] (ii) P[X2<3]

\*\*\*\*\*\*

**###Q1 .**  **Draw a random sample of size 18 from the bivariate normal distribution**

**( 2.8, 11.0, 0.16, 100,0.6).**

**Algorithm**

**i) Draw a random sample (ui,vi);i=1,2,…n from the N(0,1) distribution.**

**ii) Use the following transformation**

**xi=ui(sqrt(1+rho)) + vi(sqrt(1-rho))**

**yi= ui(sqrt(1+rho)) - vi(sqrt(1-rho))**

**This (xi,yi ) will follow Normal distribution with parameters (0,0,1,1,rho)**

**iii)Let zi and wi be the two r.v.s such that**

**zi=mu1+(xi\*sigma1)**

**wi= mu2+(xi\*sigma2)**

**(zi,wi)~BN(mu1,mu2,sigma1,sigma2,rho)**

**Solution ;**

**> u=rnorm(18,0,1);u**

[1] -1.24625371 -0.00648530 0.71622172 -0.30512269 -1.60056637 0.55544672

[7] -0.17213842 1.69416557 0.09216263 -0.78260830 0.92953477 0.65961185

[13] 1.74060379 0.04028670 -0.39147544 0.18272770 -0.52448309 0.74795585

**> v=rnorm(18,0,1);v**

[1] -1.2612821 -0.2862029 0.5397977 0.6103576 2.3830841 0.5523973

[7] 1.2103977 0.9624213 0.6126716 -1.5636976 -0.2849934 -2.2604326

[13] -0.6398013 0.4598147 -1.1047212 0.1068242 0.4103737 0.1756734

**> #By applying transformation we get random sample from biv. Normal with (0,0,1,1,0.6) d.f.**

**> x=(u\*sqrt(1+0.6))+(v\*sqrt(1-0.6));x**

[1] -2.374105e+00 -1.892139e-01 1.247355e+00 7.100621e-05 -5.173794e-01

[6] 1.051957e+00 5.477829e-01 2.751657e+00 5.040651e-01 -1.978899e+00

[11] 9.955332e-01 -5.952728e-01 1.797063e+00 3.417714e-01 -1.193869e+00

[16] 2.986959e-01 -4.038813e-01 1.057203e+00

**> y=(u\*sqrt(1+0.6))-(v\*sqrt(1-0.6));y**

[1] -0.7786952815 0.1728072861 0.5645587620 -0.7719771339 -3.5317688114

[6] 0.3532239583 -0.9832625038 1.5342801141 -0.2709100263 -0.0009606869

[11] 1.3560244399 2.2639734287 2.6063548861 -0.2398532370 0.2035054044

[16] 0.1635727220 -0.9229676088 0.8349919924

**> z=2.8+x\*0.16;z**

[1] 2.420143 2.769726 2.999577 2.800011 2.717219 2.968313 2.887645 3.240265

[9] 2.880650 2.483376 2.959285 2.704756 3.087530 2.854683 2.608981 2.847791

[17] 2.735379 2.969153

**> w=11+y\*100;w**

[1] -66.86953 28.28073 67.45588 -66.19771 -342.17688 46.32240

[7] -87.32625 164.42801 -16.09100 10.90393 146.60244 237.39734

[13] 271.63549 -12.98532 31.35054 27.35727 -81.29676 94.49920

**> #Random sample in table form**

**> data.frame(z,w)**

z w

1 2.420143 -66.86953

2 2.769726 28.28073

3 2.999577 67.45588

4 2.800011 -66.19771

5 2.717219 -342.17688

6 2.968313 46.32240

7 2.887645 -87.32625

8 3.240265 164.42801

9 2.880650 -16.09100

10 2.483376 10.90393

11 2.959285 146.60244

12 2.704756 237.39734

13 3.087530 271.63549

14 2.854683 -12.98532

15 2.608981 31.35054

16 2.847791 27.35727

17 2.735379 -81.29676

18 2.969153 94.49920

>

**> ###Q2. . Generate a random sample of size 20 from the Bivariate Poission distribution with**

**Parameter θ1=10, θ2=5, θ12=6 . Compute mean and variance for given sample.**

**Algorithm:**

**Given r.v.s follow Bivariate Poisson Distribution**

**(X1,X2) ~Bivariate Poisson(θ1, θ2, θ3)**

**Where X1 ~ Poiss(θ1+ θ3)**

**X2 ~ Poiss(θ2+ θ3)**

**And the expectation and variance will be**

**E(X1) = θ1+ θ3 = Var(X1)**

**E(X2) = θ2+ θ3 = Var(X2)**

**With correlation**

**Corr= θ3/(sqrt(Var(X1))\*sqrt(Var(X2))**

**i) Generate the 3 independent Poisson random variables Y1,Y2,Y3 with parameters θ1, θ2, θ3**

**We know that sum of the two Poisson random variables is a poisson variable**

**So accordingly, X1 ~ Poiss(θ1+ θ3) and X2 ~ Poiss(θ2+ θ3)**

**ii) Finally take a random sample from these random variables**

**Solution**

**> y1=rpois(20,10);y1**

[1] 15 9 9 11 8 10 11 6 15 8 12 12 11 9 12 10 6 10 10 10

**> y2=rpois(20,5);y2**

[1] 9 5 10 6 4 4 4 5 4 8 3 8 8 7 4 8 4 4 2 2

**> y3=rpois(20,6);y3**

[1] 4 7 10 7 3 5 3 9 4 6 4 9 4 5 6 6 7 3 4 7

**> #By applying following transformation we get random sample from biv.Poisson(10,5,6)**

**> x1=y1+y3;x1**

[1] 19 16 19 18 11 15 14 15 19 14 16 21 15 14 18 16 13 13 14 17

**> x2=y2+y3;x2**

[1] 13 12 20 13 7 9 7 14 8 14 7 17 12 12 10 14 11 7 6 9

**> #Random sample in table form-**

**> data.frame(x1,x2)**

x1 x2

1 19 13

2 16 12

3 19 20

4 18 13

5 11 7

6 15 9

7 14 7

8 15 14

9 19 8

10 14 14

11 16 7

12 21 17

13 15 12

14 14 12

15 18 10

16 16 14

17 13 11

18 13 7

19 14 6

20 17 9

**> m1=mean(x1);m1**

[1] 15.85

**> m2=mean(x2);m2**

[1] 11.1

**> v1=var(x1);v1**

[1] 6.45

**> v2=var(x2);v2**

[1] 13.77895

>

>

**> ###Q3. Generate a random sample of size 20 from the bivariate exponential distribution with failure rates λ1=3, λ2=12, λ3=10 . Compute mean and variance for given sample**.

**Algorithm**

**i) Generate the Independent Exponential Distribution with the parameters (lamda1+lamda3) and (lamda1+lamda3)**

**ii) Generate the random sample by forming the dataframe**

**where X1 ~ exp(lamda1+lamda3)**

**X2 ~ exp(lamda2+lamda3)**

**With the expectation and variance as follows**

**E(X1) = 1/ (lamda1+lamda3)**

**E(X2) = 1/ (lamda2+lamda3)**

**Var(X1)= 1/ (lamda1+lamda3)^2**

**Var(X2)= 1/ (lamda2+lamda3)^2**

**And the correlation will be**

**Corr = lamda3/(sqrt(Var(X1\*Var(X2)))**

**Solution :**

**> y1=rexp(20,3);y1**

[1] 0.08424588 0.18652908 0.12730537 0.89507253 0.30628880 0.13722707

[7] 0.16086865 0.04464543 0.50714482 0.79135867 1.08115607 0.14051731

[13] 0.25136867 0.07486680 0.20370633 0.20863776 0.09820551 1.47404787

[19] 0.12956387 0.41859667

**> y2=rexp(20,12);y2**

[1] 0.178810635 0.007543376 0.247268588 0.315560041 0.100747876 0.133674994

[7] 0.042341921 0.147026520 0.011074960 0.093114881 0.009428058 0.134739287

[13] 0.001086674 0.025460148 0.054102231 0.012764901 0.121237510 0.064872582

[19] 0.030282038 0.061697340

**> y3=rexp(20,10);y3**

[1] 0.048224702 0.165339886 0.144612521 0.039036653 0.152163818 0.185195507

[7] 0.094699229 0.040079482 0.167385967 0.004996582 0.293185869 0.249700492

[13] 0.154603638 0.062267411 0.209209929 0.136801116 0.181079389 0.022298483

[19] 0.015124884 0.399595049

**> x1=y1+y3;x1**

[1] 0.13247058 0.35186896 0.27191789 0.93410918 0.45845262 0.32242258

[7] 0.25556788 0.08472491 0.67453078 0.79635525 1.37434194 0.39021781

[13] 0.40597230 0.13713421 0.41291626 0.34543887 0.27928490 1.49634635

[19] 0.14468875 0.81819172

**> x2=y2+y3;x2**

[1] 0.22703534 0.17288326 0.39188111 0.35459669 0.25291169 0.31887050

[7] 0.13704115 0.18710600 0.17846093 0.09811146 0.30261393 0.38443978

[13] 0.15569031 0.08772756 0.26331216 0.14956602 0.30231690 0.08717106

[19] 0.04540692 0.46129239

**> #Random sample in table form is-**

**> data.frame(x1,x2)**

x1 x2

1 0.13247058 0.22703534

2 0.35186896 0.17288326

3 0.27191789 0.39188111

4 0.93410918 0.35459669

5 0.45845262 0.25291169

6 0.32242258 0.31887050

7 0.25556788 0.13704115

8 0.08472491 0.18710600

9 0.67453078 0.17846093

10 0.79635525 0.09811146

11 1.37434194 0.30261393

12 0.39021781 0.38443978

13 0.40597230 0.15569031

14 0.13713421 0.08772756

15 0.41291626 0.26331216

16 0.34543887 0.14956602

17 0.27928490 0.30231690

18 1.49634635 0.08717106

19 0.14468875 0.04540692

20 0.81819172 0.46129239

**> m1=mean(x1);m1**

[1] 0.5043477

**> m2=mean(x2);m2**

[1] 0.2279218

**> v1=var(x1);v1**

[1] 0.1578046

**> v2=var(x2);v2**

[1] 0.01365986

**> 9(B)**

**## Q4.**

**. Let (X , Y) has bivariate normal distribution with parameters μ1 = 3, μ2 = 1, σ12 = 16,**

**σ22 = 25 and ρ = 3/5. Determine the following probability.**

**(a) P(3<Y<8) (b) P(3<Y<8/ X= 8) (c) P ( -3<X<3) (d) P ( -3<X<3 / Y= -4)**

**Algorithm :**

**Solution**

> #1)

> p1=pnorm(8,1,5)-(pnorm(3,1,5));p1

[1] 0.2638216

> #2

> p2=pnorm(8,19/4,4)-pnorm(3,19/4,4);p2

[1] 0.4608732

> #3)

> p3=pnorm(3,3,4)-(pnorm(-3,3,4));p3

[1] 0.4331928

**> ##Q. 5. .Let ( X, Y) have a bivariate normal distribution with parameter μ1 = 5, μ2 = 10,**

**σ12 = 1, σ22 = 25 and ρ >0. If P (4<Y<16 / X=5) = 0.954 determine ρ .**

Algorithm :

X~N(5,1) & Y~N(10,25)

(Y/X=5)~N(10,25(1- ρ2))

P

2 P=1.954

P=0.977

**Solution :**

> rho=qnorm(0.977,0,1);rho

[1] 1.995393

**> ##Q6**

**6. The life of a tube (X1) and the filament diameter (X2) are distributed as a bivariate normal random variable with parameter μ1= 2000 hrs, μ2 = 0.10 inch , = 2500 h2 and =0.01inch2 and =0.87. The quality control manager wishes to determine the life of each tube by measuring the filament diameter. If a filament diameter is 0.098, what is the probability that the tube will last for 1950 hours? 2122**

**Algorithm**

Here (X1,X2)~N(2000,0.10,2500,0.01,0.87)

(X1/X2=0.098)~N(1999.13,607.75)

**Solution**

> #P(x1>=1950/x2)=0.098)

> px12=pnorm(1950,1999,13,24,65);px12

[1] -9.410308

**> ##Q 7**

**7. If (X1, X2) ~ Bivariate exponential distribution with failure rates λ1=3, λ2=9, λ3=10 then compute marginal distribution of X1 and X2, hence find**

**i) E(X1) ii) E(X2) iii) V(X1) iv) V(X2) v) Correlation(X1, X2)**

. .

**Algorithm**

Here (X1,X2)~Biavariate Exp(λ1=3, λ2=9, λ12=10)

Then we know,

X1~Exp(13) & X2~Exp(19)

**i)**E(X1)=1/( λ1+ λ12) E(X2)=1/(λ2+λ12)

E(X1)=1/(13) E(X2)=1/(19)

**ii)**V(X1)=1/( λ1+ λ12)2 V(X2)=1/(λ2+λ12)2

V(X1)=1/( 13)2 V(X2)=1/(19)2

**iii)** Corr(X1,X2)=

Corr(X1,X2)=

**Solution**

> lamda1=3

> lamda2=9

> lamda3=10

> meanofx1=1/(lamda1+lamda3);meanofx1

[1] 0.07692308

> meanofx2=1/(lamda2+lamda3);meanofx2

[1] 0.05263158

> varofx1=(1/((lamda1+lamda3)^2));varofx1

[1] 0.00591716

> varofx1=(1/((lamda2+lamda3)^2));varofx1

[1] 0.002770083

**> ##Q8.**

8. Suppose that an instrument has two components A and B with life time ( in minutes) X1 and X2. (X1,X2) ~ Bivariate exponential distribution (λ1=8, λ2=10, λ3=14). Calculate

i) P [first component fail before 10 minutes]

ii) P [Second component survive at least for 20 minutes]

**Solution :**

> lamda1=8

> lamda2=10

> lamda3=14

> p1=pexp(10,(lamda1+lamda3));p1

[1] 1

> p2=1-pexp(20,(lamda2+lamda3));p2

[1] 0

> ##

**> ##Q10.**

**If (X1, X2) ~ Bivariate Poisson distribution with parameter θ1=8, θ2=6, θ12=4 then compute marginal distribution of X1 and X2, hence find**

**i) E(X1) and E(X2) ii) V(X1) and V(X2) iii) Correlation(X1, X2)**

**iv) P(X1=5/X2=**

**Algorithm:**

Here (X1,X2)~P(θ1=8, θ2=6, θ12=4)

Now we know X1~P(θ1+ θ12=12) and X2~P(θ2+θ12=10)

Thus E(X1)= V(X1)=8+4=12

E(X2)=V(X2)=6+4=10

Hence as we know

Corr(X1,X2)=

Corr(X1,X2)=

**Solution**

> theta1=8

> theta2=6

> theta12=4

> meanofx1=theta1+theta12;meanofx1

[1] 12

> meanofx2=theta2+theta12:meanofx2

> varofx1=theta1+theta12;varofx1

[1] 12

> varofx2=theta2+theta12;varofx2

[1] 10

> #correlation between x1 and x2

> corr=theta12/((sqrt(theta1+theta12))\*(sqrt(theta2+theta12)));corr

[1] 0.3651484

**> ##Q11.**

. If (X1, X2) ~ Bivariate Poisson distribution with parameter θ1=3, θ2=2, θ12=8 then

Compute (i) P[X1>4] (ii) P[X2<3]

**Algorithm**

**(X1,X2)=Bivariate Poisson(θ1=3, θ2=2, θ12=8)**

**i)X1~P(θ1+ θ12=11)**

**ii)X2~P(θ2+ θ12=10)**

**Solution**

> theta1=3

> theta=2

> theta12=8

> #p(x1>)=p1

> p1=1-ppois(4,(theta1+theta12));p1

[1] 0.9848954

> #P(x2<3)p2

> p2=ppois(3,(theta2+theta12))-dpois(3,(theta2+theta12));p2

[1] 9.396275e-05

>